

B.Sc Semester II Examination 2020  
Economics Honours  
Paper: Core Course IV (CC 4)  
Mathematical Methods in Economics – II

Full Marks – 50, Time – 2 hours

**Attempt all the groups**

**Group – A**

**Answer any five questions (5x2=10)**

1. “The level curve relating to the function  $f(x, y) = Ax_1^{0.25}x_2^{0.75}$  is negatively sloped and convex.” Is the statement correct?
2. State the Duality theorem.
3. Define demarcation curves.
4. Find the dual of the following LPP.

$$\begin{aligned} \text{Maximize } z &= 2x_1 + 3x_2 + x_3 \\ \text{Subject to } 4x_1 + 3x_2 + x_3 &\leq 6 \\ x_1 + 2x_2 + 5x_3 &\leq 4 \\ x_1 + 2x_2 + 5x_3 &\leq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

5. Give the definition of homothetic function.
4. Show that all points  $(x,y)$  satisfying  $xy = 3$  lie on a level curve for the function  $g(x,y) = \frac{9(xy+1)^2}{x^4y^4-1}$
5. Is the function  $f(x_1^{\frac{1}{2}}x_2^{\frac{1}{4}})$  quasiconcave? Is it also concave?
6. Use the implicit function theorem to show that  $x^2 + 3y^2 = 25$  implies an explicitly defined function  $y = f(x)$  at the point  $(3, 4)$  and find the value of the derivative at this point.
7. Consider the function  $z = f(x,y) = x^{1/2}y^{1/3} + y^{3/2}$ . Is it homogeneous?
8. Solve  $(t+2y)dy + (y + 3t^2) dt = 0$

**Group B**

**Answer any two questions (2x4 = 8)**

11. Let  $f(x, y, a) = ax^2 - 2x + y^2 - 4ay$ . Verify Envelope Theorem. 4

12. Check Euler's theorem for the function  $f(x,y) = \frac{xy}{x^2+y^2}$  4

13. Let the utility function of A is given by  $u_A = (x + a)^\alpha(y + b)^\beta$ , while the utility function of B is  $u_B = \{(x + a)^\alpha(y + b)^\beta\}^2$ , where a, b,  $\alpha$  and  $\beta$  are all positive. Do the indifference curves of A and B have the same slope? If so, then why? 1 + 3

14. Given the demand and supply of cobweb model as:  
 $Q_{dt} = 18 - 3P_t$  and  $Q_{st} = -3 + 4P_{t-1}$ , find the time path of price and comment on its stability. 3+1

### Group - C

#### Answer any four questions (4x8 = 32)

15. i) "All homogeneous functions are homothetic but the converse is not true." Justify the statement.

Which of the following function  $f(x,y)$  are homothetic? 3+5

i)  $x^2 + y^2$       ii)  $e^{x^2y}$

16. Given the following autonomous differential system,

$$x'(t) = f(x, y)$$

$$y'(t) = g(x, y)$$

Assume that  $f_x < 0$ ,  $f_y > 0$ ,  $g_x > 0$ ,  $g_y < 0$ . Construct an appropriate phase diagram, draw the streamlines, and determine the nature of the equilibrium. 8

17. Find the demand functions for the utility function  $U = x(y + 1)$ , where the price of good X and good Y are  $P_x$  and  $P_y$  respectively and the consumer's money income is Rs. M. Hence find the indirect utility function and verify Roy's identity. 3 + 2 + 3

18. Consider the following Linear Programming Problem:

$$\text{Maximize } Z = 50x_1 + 30x_2$$

$$\text{Subject to } 2x_1 + x_2 \leq 14$$

$$5x_1 + 5x_2 \leq 40$$

$$x_1 + 3x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

Solve the problem graphically and indicate the basic solutions, basic feasible solutions and the optimum solution. 3+2+2+1

19. Find the critical points of  $f(x,y) = 3x^3 - 5y^2 - 225x + 70y + 23$  and determine if at these points the function is at a relative maximum, relative minimum, inflection point, or saddle point. 4 + 4

20. Maximize the profit function  $\pi = 64x - 2x^2 + 96y - 4y^2 - 13$  subject to the production constraint  $x + y \leq 36$  8.

21. Find the general solution  $y''(t) + 6y'(t) + 9y(t) = 27$ . 8

22. Find the general solution of  $y_{t+1} + \frac{1}{4}y_t = 5$ . Given  $y_0 = 10$  and  $y_1 = 5$ , find the complete solution.

6+2